CORRECTNESS OF ASSERTION AND VALIDITY OF INFERENCE

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As you can see from its title, my talk falls into two parts, and the order between these two parts is essential: one has to begin with correctness of assertion and then deal with validity of inference only after that. In addition to these two parts, I will add a third part in the middle about the relation between correct assertion and knowledge.

Let us begin, then, with the very notion of assertion, usually written like this,

$$\vdash C$$

Throughout, I use C for the content of an assertion, and I use the Frege sign $\vdash$, or the assertion sign in Russell's terminology, for the assertoric force. An assertion in general is just what you obtain by taking a content C and prefixing the assertion sign to it—a purely formal addition that you make. That fits with the general way assertion—påstående in Swedish—functions in grammar, namely that whenever you assert something, that is, you utter something which has the assertoric force in it, then you are held responsible for what you have said: you have made an assertion, however wrong it may be. What makes it into an assertion is merely the fact that it begins with the assertion sign.

The other part here, namely the content C, I have to say something about, because it is given a semantic definition, simply as something to do. The content is thus typically expressed by an infinitive phrase, for instance, to multiply together two arbitrarily given decimal numbers—that is an infinitive phrase, and it expresses a particular content. To swim 200 meters is also something to do, so it can likewise be the expression of a content. As a term for something to do I will generally use task, Aufgabe in German, which was introduced, albeit for propositions (Aussagen) rather than assertoric contents, by Kolmogorov in 1932.

As far as the assertion sign is concerned, this explanation was a purely formal one. From the contentual point of view, we also have to explain what is the purpose of uttering an assertion. I take it that to explain the meaning of a complete sentence is the same as explaining the purpose of the act of uttering it, that is, saying something by means of it. So the question is, What is the purpose of an utterance of the assertion $\vdash C$? To answer this question we have to introduce, not only the speaker, who produces the assertion, but also the hearer, who receives the assertion from the speaker, as indicated in the figure,

$$\text{speaker} \longrightarrow \ \vdash C \ \longrightarrow \text{hearer}$$

This is necessary, because we cannot explain the meaning, which is to say the purpose, of an assertion speaking about the speaker alone: the meaning has to do with the interaction between the speaker and the hearer. The definition of the purpose that I suggest is that the purpose of the speaker's utterance of $\vdash C$ is to permit the hearer to request the speaker to do C, whereupon the speaker becomes obligated to do C, that is, to fulfil the hearer's request. Now we have, not only the speaker's act of uttering the assertion, but also the hearer's dual speech act of requesting the speaker to do C. (To do C makes sense, because, by definition, C is something to do.)

What I have said in words in this way can be elucidated by the following diagram,

$$\frac{\vdash C \qquad C?}{C \text{ done}}$$

We have the assertion, $\vdash C$, and then have the request from the hearer, which I write with a question mark as $C?$, and as a result of that, the speaker gets obligated to do C, so C becomes done, or if you prefer, fulfilled. Now I have written it in a way that makes it look maximally like an inference rule, but you see, this is not an inference rule in the ordinary sense: it is a rule of interaction between speaker and hearer, and our ordinary inference rules are not like this at all. We can call it the assertion-request-manifestation rule,

$$\frac{\text{assertion} \qquad \text{request}}{\text{manifestation}}$$

We could also call it simply the interaction rule, regulating as it does the interaction between the speaker and the hearer.

I have already explained what the purpose of a speaker's utterance of an assertion is. I may rephrase that formulation by saying that to make an assertion, $\vdash C$, is to assume a conditional obligation, conditional commitment, namely the obligation of answering the hearer's request by actually doing C. This means that the explanation that I have provided here of assertion is the so-called commitment account of assertion: an assertion, $\vdash C$, is by definition a commitment, namely a commitment to do C in case I get a request $C?$ from the hearer.

I also want to make the observation that the assertion-request-manifestation rule has the assertion sign appearing in elimination position in Gentzen's terms: it is the major premiss, the first premiss, that carries the assertion sign, and that is precisely the place where the logical operators appear in the elimination rules of Gentzen's system of natural deduction.

With this I take it now that I have said enough, for the moment, about what assertion is. The second question after that is, What is correctness of assertion, what does correctness mean? The most well-known formulation is that the condition for an assertion to be correct is that the asserter knows C to be true ($\vdash C$ can be read simply as 'C is true'). The idea is this, that for an assertion to be correct, it is not sufficient that what is held true is true: the asserter, the one who makes the assertion, has to know that its content is true. He is responsible for the assertion.

I will accept without further ado the definition of truth as doability, or fulfillability. Then something specifically constructive comes in, namely the identification of knowing the fulfillability of C with knowing how, or being able to fulfil C,

$$\begin{array}{rl} & \text{to know } \vdash C \\ = & \text{to know that } C \text{ is true} \\ = & \text{to know that } C \text{ is doable (fulfillable)} \\ = & \text{to know how (be able) to do (fulfil) } C \end{array}$$

In this way, knowledge of truth, i.e. knowledge that in Ryle's terminology, is analyzed as knowledge how, which is characteristic of constructivism in logic. As a further side remark, I just want to observe that the notion of potentiality is involved here in the ending -able. When I speak about fulfillability or doability, I am taking the notion of possibility, or potentiality, simply for granted.

These are the first things to say about assertion, and this defines what it means for an assertion to be correct. On the other hand, that seems very limiting in an unnatural way, because correctness has a very general sense. Actions have their purposes, and we take that as an axiom: just as things have their essences, actions have their purposes. The notion of correctness makes sense, not only for assertions, but for actions in general, and correctness then has a natural interpretation as, in Swedish, ändamålsenlighet, which, unfortunately, has no single word corresponding to it in English: adaptedness, suitedness, or fittedness to its purpose. It is interesting to observe that we have good teleological terminology in German and Swedish, for historical reasons presumably, whereas in French and English you lack a term for ändamålsenlig. You could invent something like 'purpose-fulfilling', but there is no generally accepted single word. So, correctness, when we are not limited to assertions, is simply identified with purpose-fulfillingness, ändamålsenlighet.

In this general situation, when we are dealing with arbitrary actions and their purposes, the definition of correctness that I gave for assertions has to be generalized so as to say instead directly that it is correct of the speaker to make the assertion $\vdash C$ if and only if the speaker knows how to do C, or is able to do C, or simply, in the present tense, can do C. Since this is the most crucial single sentence in this talk, maybe I could write it on the board,

$$\begin{array}{ll} & \text{correct of the speaker to make the assertion } \vdash C \\ \text{(Correct)} & \text{if and only if} \\ & \text{the speaker knows how to (is able to, can) do } C \end{array}$$

Then we need to see to it that this is in agreement with how I defined correctness of assertion previously. The criterion of the speaker's ability to do C is that the speaker does C on request, that is, that he does C in case he is requested by the hearer to do so. On the other hand, the criterion of the correctness of the speaker's assertion—I have already given the general definition of correctness as purpose-fulfillingness, and then the criterion that the purpose is fulfilled also amounts to the speaker's doing C in case he gets a request from the hearer. In both cases, the criterion—of correctness and of the speaker's ability, respectively—is that he does C on request, and because of the sameness of the two criteria, I take the equivalence of the two members of the correctness principle to be established.

Now, remember the commitment account of assertion: to make an assertion $\vdash C$ is to obligate oneself to do C on request. It permits us to reformulate the principle

(Correct) in terms of obligation rather than assertion,

> correct to obligate oneself to do something on request
> if and only if
> one knows how to (is able, can) do it

which can be further contracted into

> correctly obligated
> if and only if
> can

The reason behind the contraction is that I want it to become visible that this is a biconditional version of the ought-implies-can principle, which has its origin in Roman law, but got its name, ought-implies-can, from G. E. Moore in 1922. This principle says that if you are obligated to do something, then you can do it. That means that, if it is impossible for someone to do something, then he cannot be blamed for neglecting an obligation to do it, because the obligation is waived in that situation. The biconditional ought-implies-can differs from the ordinary formulation of it, not only in having 'if and only if' instead of 'implies', but also in the addition of 'correctly' as a qualification of 'obligated' immediately before the biimplication, and it is this addition of 'correctly' that makes the implication hold in both directions.

Since in the principle (Correct) we have a purely conceptual connection between the three concepts of assertion, correctness and ability, the instinct is of course to try to define one of the three concepts in terms of the others, so as to make the principle itself valid by definition. Timothy Williamson's view of correctness is that correctness of assertion is defined in terms of knowledge, and when I speak about knowledge here it will always be about knowledge how. That puts the order that knowledge is prior to correctness, but there is another possibility here, namely to define knowledge as correct assertion. Interestingly enough, that does occur in the literature, namely in § 36 of Bolzano's *Wissenschaftslehre*, a short paragraph in which Bolzano simply defines knowledge as correct assertion, Erkenntnis gleich richtiges Urteil in the German. It is not an easy matter to make up one's mind about the order of conceptual priority between knowledge and correct assertion, but as should be clear from the preceding part of this talk, I have come to side with Williamson.

To summarize the comparison with Williamson I would say that my analysis could be characterized as an equation,

	commitment account of assertion
+	knowledge account of correctness (of assertion)
=	total account of correct assertion

This formula is to be compared with Williamson's expression 'knowledge account of assertion', which makes it sound as if the total account of assertion were in terms of knowledge. We have to divide it up into two parts. The definition of assertion, which I began with, is in terms of obligation, or commitment—the definition of assertion is not in terms of knowledge. It is the correctness of assertion that brings in the knowledge component. This is a summary of my view of it and which makes it clear how it compares with Williamson's view.

With this I have finished the second part, on the relation between correct assertion and knowledge, and it is time to go over to validity of inference, which is the title of our symposium. The first formulation that comes to my mind, and that one cannot avoid in one form or another, is that an inference is valid if and only if, given that the premiss assertions have been correctly made, it is also correct to make the conclusion assertion. I am considering an inference of this schematic form,

$$\text{(Inf)} \qquad \frac{\vdash C_1 \;\ldots\; \vdash C_n}{\vdash C}$$

and the formulation I used is that this rule is valid if and only if, given that the premiss assertions $\vdash C_1, \ldots, \vdash C_n$ have all been correctly made, it is also correct to make the conclusion assertion, $\vdash C$. Using the terminology of preservation, we may express this more briefly by saying that the rule (Inf) is valid provided it preserves correctness from the premisses to the conclusion.

Yet another way of formulating the definition of validity of inference is to introduce the metalinguistic counterpart of (Inf),

$$\frac{\mathrm{cor} \vdash C_1 \;\ldots\; \mathrm{cor} \vdash C_n \qquad \mathrm{val}\left(\dfrac{\vdash C_1 \;\ldots\; \vdash C_n}{\vdash C}\right)}{\mathrm{cor} \vdash C}$$

and take it to be meaning determining for the last premiss.

The rule (Correct) now permits us to reformulate the preceding three formulations of the definition of validity so as to be expressed in terms of knowledge rather than correct assertion. The condition for the rule (Inf) to be valid is that, once all of the premisses have become known, also the conclusion gets to be known. In terms of preservation, this is to require that knowledge is preserved during the passage from the premisses to the conclusion. Finally, in terms of knowledge rather than correctness of assertion, the metalinguistic version of (Inf) takes on the form

$$\frac{\mathrm{known} \vdash C_1 \;\ldots\; \mathrm{known} \vdash C_n \qquad \mathrm{val}\left(\dfrac{\vdash C_1 \;\ldots\; \vdash C_n}{\vdash C}\right)}{\mathrm{known} \vdash C}$$

Also in this epistemic form, it may be said to be meaning determining for the last premiss: the change from correctness to knowledge only affects the first n premisses and the conclusion.

I would like to end by briefly considering inference and its validity from the point of view of dialectical logic. This idea of bringing in the dialectical interpretation of logic in connection with inference is due to Göran Sundholm. He used the formulation,

> When I say 'Therefore', I give others my authority for asserting the conclusion, given theirs for asserting the premisses.

This is the first place I know of where the dialectical perspective in logic, pushed by Lorenzen and his collaborators from 1958 and onwards, has entered in connection with inference. I heard this from Sundholm back in 2009, I believe, and then I did not react very much. I thought it was a clever paraphrase of Austin's "Other minds", which indeed it is, but it could be serious even if it is easy to take it as more of a pun: in an inference we have a passage from others who have given to you the premisses, and then you trust them on their authority, and on the other hand, you, who are the receiver of these premisses, give authority to the conclusion, which may be received by someone else further on.

I would like to rephrase Sundholm's definition so that, without becoming directly equivalent, it nevertheless brings out the dialectical, or interactive, character of inference in a more explicit way. Here is my variation on Sundholm's formulation. The concluder receives the premisses from the premissers, and in turn gives away, or passes on, the conclusion. As a result of receiving the premisses from the premissers, the concluder gets the right, or permission, to request the premissers to perform their respective tasks $C_1, \ldots, C_n$. The validity of the rule is tantamount to the concluder's ability to perform C when given this help from the premissers. Thus the effect is that the premissers together with the concluder can perform the conclusion task C. You see the novelty that is not present in the usual explanations of inference and rules of inference: the novelty is that in an inference, the concluder gets the right to ask the premissers to perform their respective tasks $C_1, \ldots, C_n$, and that means that the concluder gets helped by the premissers to perform these tasks. The validity of the rule is tantamount to the concluder's being able to do C given this help from the premissers to do $C_1, \ldots, C_n$. This is a less elegant formulation, but it has the advantage of bringing out the interactive character of inference more clearly than Sundholm's formulation.