

ASSERTION AND REQUEST (STOCKHOLM LECTURE)

PER MARTIN-LÖF

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I would like to start from something very well known, namely the contrast between two conceptions of language: the old-fashioned one, that language is defined as the expression of thought, and the modern conception of language as a means of communication. The reason why I bring it up is that it is connected with another distinction which is important for this talk, namely the contrast between the monological versus the dialogical conception of logic. Why are they connected? Well, if thought indeed is monological, simply because it is something that goes on in our minds, and other people, at least in the present state of knowledge, do not have access to that, hence it is by its very nature monological, then if language is defined as the expression of thought, this monological character is reflected also on language, whereas if language is a means of communication, then it is so to say dialogical, not monological, from the very beginning.

The first time I personally was confronted with the dialogical conception of logic was in Aarne Ranta's book *Type-Theoretical Grammar*, where he gives two examples which go beyond the traditional view of logic as the theory of assertion and inference. Logic is traditionally defined as the study of reasoning, and it does not matter if you say reasoning or inference there, but that definition is defective in the sense that you have to introduce the concept of assertion first, because the premisses and the conclusion of an inference are assertions. So, the traditional definition could be phrased by saying that logic is the theory of assertion and inference.

Let me give Ranta's two examples, which go beyond this traditional view. Suppose we are in constructive logic. Then, if someone makes the assertion that a disjunction is true,

$$\vdash A \vee B \text{ true}$$

then someone else has the right to challenge this assertion of his:

$$? \vdash A \vee B \text{ true}$$

When confronted with that challenge—let us call the agent of the challenge the opponent and the original assertor the respondent—then the opponent has the right to challenge the assertion of the respondent, and as a result of that, the respondent

becomes compelled to answer either that A is true or that B is true.

$$(Dis) \quad \frac{\vdash A \vee B \text{ true} \quad ?\vdash A \vee B \text{ true}}{\vdash A \text{ true} \mid \vdash B \text{ true}}$$

In constructive logic this rule clearly seems to be valid, because in constructive logic you can do this, you can make the choice between these two. You can see that we are forced to introduce some new notation here because (Dis) is a kind of rule which is not an inference rule in the usual sense in logic, because an inference in the usual sense has a finite number of premiss assertions and a single conclusion, which is also an assertion, whereas here we are faced with something that does not show that pattern. What is particularly important here for the following is this interplay between rights and duties, or permissions and obligations, namely that $\vdash A \vee B \text{ true}$ is an ordinary assertion, so no problem, but in $?\vdash A \vee B \text{ true}$ we have the opponent, by the assertion, getting the permission to challenge it, right, or permission, to challenge it, and as a result of that challenge, the assertor, or respondent, is obligated, he must, respond by either asserting A true or by asserting B true. So there is an interplay here between rights and duties.

The other example that Ranta gave is the similar one for existence.

$$(Ex) \quad \frac{\vdash (\exists x : A)B(x) \text{ true} \quad ?\vdash (\exists x : A)B(x) \text{ true}}{\vdash a : A \quad \vdash B(a) \text{ true}}$$

If someone claims $\vdash (\exists x : A)B(x) \text{ true}$, and it is challenged, then the respondent is compelled to answer, now no longer with a single assertion, but with two assertions: he has to answer with an individual in the individual domain and also with an assertion that the particular instance of the propositional function is true. Again we have a right that the opponent has and an obligation for the respondent. There is a may hidden in the challenge, $?\vdash (\exists x : A)B(x) \text{ true}$, and there is a must in the conclusion.

These were the two examples by Ranta, and one can give the same treatment for the other logical operations. The most complicated case is the implication, which requires the introduction of proofs in the sense of proof-objects, and I will avoid that as far as possible, so let us stay with these two examples. But if you take conjunction, for instance, you can immediately do it yourself following the same pattern. The connection with Gentzen's dictum that the meaning of a logical operation is defined by its introduction rules is clear here, namely that what the assertor is compelled to give are the premisses of the introduction rule that the main premiss is the conclusion of. So there is a way of passing from the introduction rules to these defence rules, back and forth.

I want to give a non-logical example of this pattern, and the reason is that that can give us a feel for what is really at stake here, what it is about. For this non-logical example, think of a child, of 5 years of age or something, running to his mother saying, Mum, I can swim! Then the mother, maybe somewhat incredulously says, Oh, can you?, in which case it is a question, or she could say, Oh, show me!, in which there would be an exclamation mark instead. That is the second step,

corresponding to the second premiss, and as a result of this the child swims. This conclusion is no longer a linguistic act, as all the other acts given here are, but the whole inference, if one calls it an inference, has the character of what Aristotle called a practical syllogism—we could say practical inference, perhaps—because the conclusion is an action rather than an assertion or some other kind of linguistic act. One of the examples given by Aristotle of a practical inference is—you have as major premiss, All sweet things ought to be tasted, or must be tasted, and then as minor premiss, This particular thing is sweet, and then the conclusion is the actual tasting of it.

The swimming example clearly brings out that this is something that has to do with knowledge how—knowledge how to swim, in this particular case—or abilities, or skills. The example shows how a particular ability is tested, we may say, I mean, the mother tests the son by saying, Oh, show me! The similarity I noted with the logical example is sufficiently clear, I hope, which makes it plausible that this all has to do with the concept of knowledge how, in Ryle’s term. I will not make any distinction between knowledge how and ability: I will treat them as the same in this talk. This talk of mine could perhaps be described as a talk about the logical manifestation of the concept of knowing how, the concept of ability.

I referred to Aarne Ranta because that is how I came into contact with this kind of logical pattern, but of course, he was not the first at all. Coming from Helsinki, he was well acquainted with Hintikka’s work, in particular Hintikka’s work in game-theoretic semantics, and he also had as a source of inspiration a beautiful paper by the Finno-Swedish philosopher Erik Stenius called “Mood and language game”, from 1967. As far as I know, there was no direct influence from the Lorenzen school. Of course, it is Lorenzen and the Lorenzen school which should be mentioned here in the first place, because it was Lorenzen who introduced into logic this dialogical conception, and also the term dialogical logic for this kind of investigation, back in 1958, and after him there has been a flourishing school in this area.

I mentioned, as a point of history, Lorenzen, who initiated it. Hintikka’s work came in the 1970s, but then during the last 20 years or so, it has been a really flourishing, exploding area, and that is perhaps witnessed best of all by Van Benthem’s founding of a series of monographs, *Texts in Logic and Games*, in 2007, with the first volume called *Interactive Logic*, published in 2008. Interactive logic is a very good term, to my mind, as an alternative to Lorenzen’s dialogical logic. The title *Logic and Games* of course indicates that the phenomena that are involved are analyzed in game-theoretic terms.

What is it that we are faced with here? Well, first of all, a new kind of speech act, in addition to assertion, namely the speech act that we see in $? \vdash A$ true and that I have chosen to call request in the title of my talk: the opponent requests the speaker to respond to the challenge. You could use question instead—the opponent questions the assertion of the respondent—or you could say challenges the assertion of the respondent. That is a new kind of speech act, which means that we have to give some account of this new kind of speech act, in addition to the account that we presumably have already given of assertion.

But there is not only this new kind of speech act: there are also the new rules of interaction, of which we have seen examples here, including the swimming example. If you call this new kind of speech act request, then it is natural to call these rules that involve the request the request rules. In (Dis) and (Ex) we have two examples of request rules. I want to make clear here that in each of these cases there is not only one rule, but there is a step already from $\vdash A \vee B$ true to $? \vdash A \vee B$ true, which I could write like this:

$$(Req) \quad \frac{\vdash A \vee B \text{ true}}{? \vdash A \vee B \text{ true}}$$

If we have an assertion, then the opponent is permitted to question it. So we have also this rule which has to come before the application of (Dis). This rule is in one sense more like an ordinary inference in logic, namely in that the opponent is permitted to do what is shown in the conclusion, just like in ordinary logic, when you have finitely many premisses, you are permitted to draw the conclusion. There is this place already in ordinary, monological logic—let us call it that—there is this place for the notion of permission, for may, in the formulation of the usual rules of inference. The first place that I know of where this deontic formulation of the ordinary rules of logic is completely explicit and clear is in the first volume of Frege’s *Grundgesetze*, so 1893, but the historical investigation of such matters is a bit difficult, so it may be that one could find it earlier, but at least I know of no earlier place than Frege in this case.

So we have really two rules here where request is involved: (Req), and then after that comes (Dis). Either you could call both of these request rules, because request is present in both of them, or you could say that (Req) is the request rule which permits you to make a request, and (Dis) is the granting rule which shows you how to grant the request that is made in its second premiss.

Now, something in general about the structure of the speech act. When we abandon the monological view and go for the dialogical, or interactive view, it means that we have in addition to the speaker also someone who hears, or reacts, to what the speaker says.

$$\text{speaker} \longrightarrow \text{sentence} \longrightarrow \text{hearer}$$

The speaker produces a sentence, a complete sentence, or utterance, and that sentence is taken up by the hearer. The monological conception naturally makes us forget the part

$$\longrightarrow \text{hearer}$$

because the receiver of the sentence is the speaker himself, hence since there is only one person involved, it does not really matter so much with the part

$$\text{speaker}$$

either, so what we are faced with is the speech act itself

$$\longrightarrow \text{sentence}$$

where the arrow indicates the act and the sentence by definition is the complete utterance, what is uttered by the speaker, what is called the complete sentence in ordinary grammar. The notion of complete sentence is standardly defined by saying that it is the smallest unit of speech which it makes sense to utter, that is, by means of which we can perform a speech act. I know of no better definition than that of the notion of complete sentence.

We now have to go into the inner structure of the sentence. We have the act of uttering the sentence, and the outermost structure of the sentence is its mood/content structure.

→ mood content

This view in grammar is above all associated with Bally, Charles Bally, a Swiss linguist, successor of Saussure at Geneva who precisely took this structure, although he used the Latin terminology

→ modus dictum

—he took the Latin terminology from the Scholastics, and his view was that this was the most basic structure of a complete sentence. That is of course in sharp contrast with the standard view that it is the subject/predicate form, and in the modern terminology of transformational linguistics, the noun phrase + verb phrase, NP + VP, form, but that is just a jazzed up subject/predicate form—so the traditional view, of course, is that the subject/predicate form is taken to be the fundamental structure of the complete sentence. Although Bally is little known, as it seems, I think at least for logic, for logicians, it is his view that we are supporting in logic.

So, two components here: the mood and the content, and both may be varied. The first mood is of course the assertive mood, assertoric or assertive mood, but others are of course the mood for a command or a question or a wish, and the list can of course be made very, very long. The manifestation of the mood in natural language is in the natural language moods, which is a very, very short list. I mean, in most of our languages it is only three that are mentioned: the indicative, the imperative and the subjunctive, and in some languages, like Greek, you also have a special mood for wishes, or prayers, namely the optative mood, so then there are four. But of course, this is a very limited number compared to the large variety that we have of different kinds of speech act.

This is the outermost structure of the sentence, and it is a striking fact that this structure is formally the same as the structure of thought, which was made particularly clear by Descartes in his *Meditations*. He distinguished between the different kinds of thought that we can have, so that means varying the mood, but also, when thinking there is something we are thinking about, there is an object that we are taking an attitude against, whether it is assertion or wish or whatever it is. The difference is that we are more used to speaking of the object of the thought as what we are thinking about, whereas I am using the term content here instead. The reason is of course that the word object in this sense is the scholastic understanding of the term object, whereas now when we speak of objects, we speak for instance of mathematical objects and physical objects and so on, and that is a

meaning of the term object which is different from the scholastic one. That is why I have chosen here to use content instead.

The fact that the structure of the *cogito*, that is, the structure of thought, is the same as the structure that we discovered by studying language, if you want the structure of the *dico*—*dico* for I say, rather than *cogito* for I think—that is a very strong argument to my mind for the parallelism of thought and language, without taking any stand on what is prior to what, if thought is prior to language—the traditional view—or if language is prior to thought—the more modern view.

The question is, How are we going to explain the two components here, the mood and the content? Concerning the content I have a proposal here, namely to—because the notion of content, assertoric content, is so closely related to the notion of proposition, one may say that a proposition is an assertoric content made into an object of your theory: made into an object because we are saying that things are propositions, we are using the form of assertion

A is a proposition

That is what I mean when I say that propositions are treated as objects, so they appear in the subject position here. Contents are not treated as objects of your theory, but because of this similarity between the notion of proposition and the notion of content, I propose to simply take the well-known constructive understanding of the notion of proposition and transferring it to the notion of assertoric contents.

As we know, in the Brouwer–Heyting–Kolmogorov interpretation, the BHK-interpretation, a proposition is interpreted either, in Heyting’s terms, as an expectation or an intention, or else in Kolmogorov’s terms, as a task, if I use task as translation of his *Aufgabe*, *uppgift* in Swedish. The difference there between the first and the second way of phrasing it is just a question of what terms you chose: the concept is one and the same. My proposal is to accept this also as an explanation of what a content is, namely that a content expresses an expectation or an intention, in Heyting’s terms, or a task, in Kolmogorov’s terms. The difference is only that it is not objectified in the theory, as a proposition is.

If we forget about the origin about my proposal and were to state it outright, then I would say that a content we should think of as simply something to do. That is what the task, or *Aufgabe*, refers to: something to do, if you phrase it actively, or something to be done, if you phrase it passively.

Let that be enough about my explanation of the content. Then we are faced with the other component, namely the mood. The mood is defined—and here there are no alternative choices, as far as I know—the mood is simply the kind of speech act. We had examples of various kinds before, and among these various kinds we have—and usually we take that in the first place—the assertoric mood. I am going to use Frege’s assertion sign for the assertoric mood, although that does not match exactly Frege’s way of using it, but I will ignore that point, and simply allow myself to use the Frege sign for the assertoric mood in general.

\vdash = assertoric mood

The question is then, Having given an explanation of the notion of content, what is the explanation of the assertoric mood? On this point I want to—so, there are many, many different views on this: what is it that makes something an assertion? Since it is the mood that makes it an assertion, there are as many different proposals for the one as for the other, and here there is no chance for me to discuss all the many proposals, so instead I want to refer you to Peter Pagin’s article about assertion in the *Stanford Encyclopedia*, which covers the most important proposals that have been given and gives an impression of how huge the literature is on this subject. I will touch upon two proposals here.

One is the view that the assertoric mood is explained by giving the conditions under which you have the right to make an assertion. Taking that condition to be that you know how to perform—we have the content C , then by laying down that the condition under which you have right to utter

$$\vdash C$$

is that you know how to do C —and what it means to do C is already fixed, because C is something to do, by my definition of content.

This is my first proposal, and this is a version of what Williamson calls the knowledge account of assertion. First of all, this very expression, account of assertion, comes from Williamson. It does not mean anything different from explanation—account means only explanation—but it is a terminology that has become generally accepted and which I am going to use. It is a knowledge account because the stipulation that I gave was that in order to have the right to make this assertion, $\vdash C$, you have to know how to perform the task, or fulfil the expectation, or intention, that is the content C . So you have the appearance of know how there, although because I identified knowing how with being able to, I could just as well have said to be able to, but then it would have been less clear that this is a version of what Williamson calls the knowledge account of assertion.

Another proposal goes back to Peirce, 1902-3. Peirce proposed that we should think of an assertion as the taking on of a responsibility, the responsibility for the content of the assertion, then. Responsibility there could just as well be replaced by obligation or duty, so it is the taking on of an obligation or of a duty, which means that the explanation as such is a deontic one rather than an epistemic one, as the first one I proposed. You could phrase it in the following way, that by making an assertion, that is, by uttering

$$\vdash C$$

the speaker undertakes the obligation, or duty, of performing C at the request of the hearer.

This explanation is clearly different from the first one that I gave. One of the reasons why I prefer the first one is that—first of all, both have to be validated, both have to be correct. I mean, in the first case, that the condition is that you know how to do C , that fits with all experience in game semantics, that what gives you the right to make an assertion is that you have a winning strategy in the game, and a winning strategy in the game, that is just the game-theoretic way of stating

that you have a method, or a way, of doing C . So, there can be no doubt that this has to be so. Similarly, I do not think there is any doubt about Peirce's proposal here in the sense that we are undertaking an obligation when we are making an assertion. The only question is, Is it the one or the other that is to be taken as the definition of assertion, the other one being a consequence of what we have chosen to take as the definition?

The argument I have for taking the account in terms of knowledge how as the fundamental one is that if we take Peirce's account as the definition of assertion, then we need a principle to go from that to the knowledge account, and the principle we need is what G. E. Moore has called the ought-implies-can principle. Since ought is ambiguous, and I do not mean ought in the sense of desirability here, you could say must-implies-can. Such a very short formulation—it is good because you can produce the name very easily, but it is ambiguous, it can be understood in many different ways, and the particular way in which it is needed here is as the principle that in order to have the right to undertake an obligation, you must be able to fulfil it, or as an imperative, See to it that whenever you undertake an obligation, you know how to fulfil it! By means of that principle, the knowledge-how formulation follows from the responsibility, or obligation, formulation that Peirce gave.

Although the connections are exactly as I have explained them here, it seems somehow intuitively implausible that you should need the ought-implies-can principle in order to get to this very basic fact that to have the right to make an assertion, you have to be able, or know how, to fulfil its content. So, I opt for the knowledge account, or know-how account, on this point.

Concerning request I hardly need to say anything, because what a request is is clear from the rules. The first rule for request, remember, was

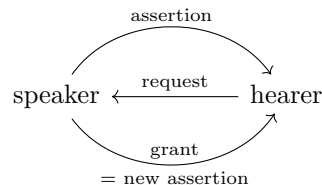
$$\frac{\vdash C}{? \vdash C}$$

This is the request rule, which is simply to say that, when someone has made an assertion, you always have the right to question it, or challenge it, by making a request. That is the first rule. The other rule, the more important one, is

$$\frac{\vdash C \quad ? \vdash C}{\vdash C'}$$

If you have an assertion, and then it is challenged, then the assertor is forced to respond to that by making some new assertion, which I indicate by $\vdash C'$, and this need not be a single one, it can even be empty here, and there can be more than one, as we saw in (Ex). And $\vdash C'$ is simply the—if we speak in terms of expectation and intention here, then it is what fulfils the expectation, or intention, C .

This pattern that I have indicated by means of these rules has another indication, which I just need to show very briefly:



We have first the assertion that is made, and then we have the hearer that responds by a request, and then the speaker responds by granting the hearer's request, and that is the new assertion $\vdash C'$. Then the dialogue can continue by the opponent's questioning the new assertion. This is just another way of symbolizing the request rules.

Let me finish then by saying something of the relation of this, what I have said today, which involves the deontic notions of right and duty, or permission and obligation, and what is called deontic logic. We have a sub-area of logic which is called deontic logic, and which goes back to Ernst Mally, a pupil of Meinong, in the 1920s, who produced the first formal system of deontic logic, and then the subject was taken up by Von Wright in 1951, and Von Wright's work is much more well known. How is what I have been doing here, which indeed involves the deontic notions of permission and obligation, or rights and duties, how is it related to deontic logic? In deontic logic the deontic notions of permission and obligation appear in the content of the assertions, or judgements, that is, in the C . On the other hand, deontic logic deals only with assertions, just like in standard logic: it is only assertions whose content contains the deontic operations. This puts an opposite order between the deontic notions and the epistemic ones from the one that I have been advocating here, namely that the deontic notions make their appearance in the analysis of what it means to know how to do something: that is where the deontic notions have their place. So, the deontic notions are at a more basic level than the epistemic notion of knowledge how, whereas in deontic logic it is the other way around: by putting the deontic notions in the content and retaining the ordinary assertoric force, which is the epistemic reading here, the deontic notions are subordinated instead to the epistemic notions. What I have advocated here, you could say, is a deontic basis of ordinary assertoric logic, rather than basing deontic logic on ordinary logic. Maybe this is the explanation for the difficulties of deontic logic: I mean, you have formal rules, but to convince yourself that these formal rules make good sense is a notorious problem, and maybe it can be explained in this way, that one has got the order of priority between the deontic notions and the epistemic notions the wrong way around.