ASSERTION AND REQUEST
(OSLO LECTURE)

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I shall take up criticism of logic from another direction, namely the criticism that you may phrase by saying that traditional logic does not pay sufficient attention to the social, intersubjective, or interactive character of language. So I bring in from the start here the two conceptions of language: one as the expression of thought, and the other—if you ask what language is for, then one answer is that language is for the expression of thought, and the other is that language is for communication. It is roughly correct to say that the first is the traditional conception, which dominated up until the last century, when it was replaced by the modern conception. We have thus this distinction between, on the one hand, the social character of language, and on the other side, the non-social view of language. A pair of words that fits very well here is to speak of the monological conception of logic, or language in general, versus a dialogical one. Here I am showing some special respect for Lorenzen, who is the one who introduced the very term dialogical logic.

The first time I was confronted with something of this sort was when reading Aarne Ranta's book *Type-Theoretical Grammar* in 1994. Ranta there gave two examples, which I will show immediately. The first example is in propositional logic, and moreover, we take it to be constructive propositional logic, because that does matter here, since the rule that I am going to show is valid constructively, but not valid classically. Suppose that someone claims a disjunction to be true, asserts, or judges, a disjunction to be true. Then someone else has the right to come and ask him, Is it the left disjunct or is it the right disjunct that is true? There comes an opponent here, who questions the original assertion, and I could write that in this way:

$$? \vdash A \vee B \text{ true}$$

By doing that, he obliges the original assertor to answer, that is, to assert, either that A is true or that B is true. So he has a choice, and we need to have some symbol for the choice here,

$$\text{(Dis)} \qquad \frac{\vdash A \vee B \text{ true} \qquad ? \vdash A \vee B \text{ true}}{\vdash A \text{ true} \mid \vdash B \text{ true}}$$

This is clearly a valid rule. It is not a rule of inference in the usual sense, but it is a valid rule for constructive propositional logic.

So, what is this? I mean, we know what the rules of propositional logic are, and here we have a simple rule which is not among the standard rules of propositional logic, so propositional logic has some kind of incompleteness here, of a kind that we are not used to. What is particularly interesting is of course the novelty—I said that if

$$\vdash A \vee B \text{ true}$$

has been asserted, then that *may* be questioned by someone. There we have a may, and that is not a big novelty, because in every standard rule of inference, say of the form

$$\text{(Inf)} \qquad \frac{J_1 \ldots J_n}{J}$$

—so, some assertions have already been made, say, and the rule says that we may proceed to assert the conclusion. That is very explicit in Frege's formulation of the inference rules in the first volume of *Grundgesetze*. It is the earliest place that I know of where the rules are systematically formulated in deontic terms like this.

Then, even more interestingly, the original assertor *must* answer either

$$\vdash A \text{ true}$$

or

$$\vdash B \text{ true}$$

So we have also a must here, which means that there is an interplay between rights and duties, or permissions and obligations, that we do not have in traditional logic, where all rules are of the form (Inf).

Ranta's second example is from predicate logic, but it is of the same kind. Someone asserts an existence statement,

$$\vdash (\exists x : A)B(x) \text{ true}$$

and then someone else comes and questions that:

$$? \vdash (\exists x : A)B(x) \text{ true}$$

In that case, the original assertor is forced, which is to say, he must come up with an individual from the individual domain and also assert that the predicate B is true of that instance,

$$\text{(Ex)} \qquad \frac{\vdash (\exists x : A)B(x) \text{ true} \qquad ? \vdash (\exists x : A)B(x) \text{ true}}{\begin{array}{c}\vdash a : A \\ \vdash B(a) \text{ true}\end{array}}$$

This is entirely similar to (Dis), but now there is no choice involved, so no vertical bar. Instead we have two assertions that the original assertor must continue to defend. The dialogue may then go on from these instead.

So, same situation here—it is no longer in propositional logic, but in predicate logic, which we are equally familiar with. It is, however, not a rule of ordinary predicate logic. It is a dialogue rule for predicate logic, which is clearly valid when the existential statement is given the constructive interpretation.

These are the two Ranta examples which were my original motivation. I also want to give an example of a very different sort, which is perhaps no longer logical. Think of the following situation. A child comes running to its mother, saying, Mum, I can swim! The mother answers, Oh, show me!, or, Oh, can you? It does not matter really if it is in the imperative mood—Oh, show me!—or if it is in the interrogative mood—Oh, can you? Both have the same effect in this situation. That is the analogue of the second step above,

$$? \vdash (\exists x : A)B(x) \text{ true}$$

So

$$\vdash (\exists x : A)B(x) \text{ true}$$

corresponds to the original claim by the child to be able to swim, and then comes this question from the incredulous mother, and then we have a conclusion, which in this case is that the child actually swims. This immediately shows that there is a similarity here with the so-called practical syllogisms of Aristotle, namely that the conclusion is not a further assertion, but the conclusion is an action. Aristotle has the example with sweets: sweet things ought to be tasted, this thing here is sweet, and then the conclusion is the actual tasting of it. These new inferences here have a similarity with practical inference, because there is a practical element involved in the conclusion in both cases: there is something that the original assertor has to do.

This example, the swimming example, which is no longer logical, seems to indicate something here, namely because swimming is knowing how, or an ability, and also in (Dis) there is an ability that is involved, namely the constructive mathematician can make the decision between A true and B true. Similarly in (Ex), he can come up with an instance, and that is a practical step, to actually provide this instance. So there is something to be learned from this second example, namely that maybe this has to do with knowledge how in general and the rules that are in play when we manifest our knowledge how to do something.

I already said that it was Ranta's book from 1994 that aroused my interest in this. Ranta came from Helsinki to study in Stockholm, so he knew very well Hintikka's work in general, and in particular Hintikka's work on game-theoretical semantics. Moreover, Ranta was inspired by Erik Stenius's beautiful article on mood and language game from 1967, which contains Wittgensteinian ideas in relation to the phenomenon of mood. Before Hintikka, there was Lorenzen, and it is really Lorenzen who has initiated the present very widespread interest in dialogue games of this sort. Lorenzen's work goes back to a short paper from 1958 with the title "Logik und Agon", so logic and contest, I guess, is the most natural translation, and that is the beginning of it. It is quite an incredible situation: with all the interest in dialogue and in dialectics in Greek philosophy, in Plato in particular, why did no one ever get interested in dialogues from a logical point of view until 60 years ago, roughly?

What are the new things that we are faced with here? Well, first of all, we have a new kind of speech act, which is performed by the—oh, I have not said that—of

course, I will use the standard terminology here: either speaker and hearer, or else respondent and opponent, or proponent and opponent, as Lorenzen usually says. The novelty is that we have a new kind of speech act in addition to assertion. Assertion we had in $\vdash A \vee B$ true already, but in $? \vdash A \vee B$ true we have a new kind which, as you have seen in the title, I have chosen to call request. There are other possibilities, of course: question is an obvious other possibility. In the conclusion, it is an assertion again, so there is just one new kind of speech act here: request. Then we have new rules of inference—if we can call them inference rules. I would be interested in having suggestions here, can we call these inference rules? I tend to think, if we generalize inference sufficiently, we can, but (Inf) is definitely what an inference rule is in the usual sense. If we call them rules of interaction, then there is at least no problem with introducing strange terminology, so let us call them rules of interaction, in addition to inference rules in the usual sense, which of course remain in place as we are used to them.

So, a new kind of speech act, and that means that I have to say something about speech acts in general. The structure of the speech act is that we have a speaker who produces a sentence, or utterance, and that gets taken up, or is heard, by the hearer:

$$\text{speaker} \longrightarrow \text{sentence/utterance} \longrightarrow \text{hearer}$$

Let us concentrate on the speech act only. We forget about who the speaker is, and we forget about the uptake by the hearer and just concentrate on this,

$$\longrightarrow \text{sentence/utterance}$$

The sentence I take to be fixed in the usual way, namely that a sentence is defined as the smallest unit of speech by means of which you can say something—a definition used by Dummett, for instance, in his Frege book. I cannot do better on that point.

So the question is, What is the inner structure of the sentence, the outermost inner structure of the sentence? I will take that to be the mood/content structure.

$$\longrightarrow \text{mood content}$$

We have two parts: the mood and the content. The content is saturated, of course, but the mood is unsaturated and needs a content to operate on to produce a sentence. I am consciously not using Frege's word force here, and the reason is that Frege's force is rather the combination

$$\underbrace{\longrightarrow \text{mood}}_{\text{force}}$$

It is Hare who has the merit of having seen that there are three components involved here, and not just two components, force and content, as Frege had. In Hare's terminology we have the neustic ($\longrightarrow$), the tropic for tropos, modus, and phrastic for the content.

This is the structure arrived at from the point of view of logic. If you look at it from the point of view of grammar, then this amounts to saying that it is not the subject/predicate form, or the NP + VP form, which is the basic form of

the sentence, but it is rather the modus/dictum structure. The subject/predicate form is the outermost form of the content, but that is below the mood/content structure and is only one part of the mood/content structure. Claiming that this is the outermost form of the sentence in ordinary languages in linguistics is, as far as I know, due to Bally in 1932. Bally was Saussure's follower, I mean he took over the chair in Geneva after Saussure, and in his book in linguistics from 1932 he starts from this modus/dictum structure. This is a remarkable thing, that what you arrive at from the point of view of linguistics is the same as what you arrive at from the point of view of logic. It makes one quite confident in this structure.

The two parts here, the mood part and the content part, can be varied independently. To get assertion we should put in the assertoric mood there, and since we now have the Frege sign, the assertion sign—that is one way of using it, it does not agree completely with Frege's way of using it, which varies as you know, but for us now I think this is the best sign for assertion, and then followed by a content, which I will use C as a schematic letter for,

$$\vdash C$$

This then is the general form of an assertion, and what the assertoric mood corresponds to, above all, in natural language is the indicative mood,

$$\text{ind } C$$

I will not spend any words on the relation between the logical moods and the linguistic moods, but surely, there are very few linguistic moods in most of our languages presently—not Finnish, but—three usually, and that is much, much less than the logical moods that you may be interested in considering. We have already two here, assertion and request, and there are wishes and fears and questions and commands and all this, already many more than three. If we want some other mood here, we simply change this to, say, the optative mood,

$$\text{opt } C$$

which is what you have in Greek to express a wish, and any of the other moods that I mentioned here. If it is the wish here, then

$$\longrightarrow \text{opt}$$

is the act of wishing and

$$C$$

is the—in the scholastic use of the term object, it is the object of the wish, that is, that which is wished, but that is what I have decided to call content here, so object or content.

So much about speech acts in general and the mood/content structure. What I meant by saying that—by bringing up the linguistic case in addition to the logical case is that if you take the logical case here and you stick to the old-fashioned view that logic studies thought primarily and language only secondarily, then the similarity of structure here means that the structure of thought is the same as the

linguistic structure: the structure of the *cogito* on the one hand and the structure of the *dico*, if you want—*dico* for I say—on the other. That seems to me to be the strongest argument that we have for the parallelism between thought and language, without going into the question of the priority between thought and language.

How are we now going to explain the components that are involved here? Let us begin with the content. I have a suggestion to make here, namely to provide a definition, or explanation, of what the content is. It is well known that there is a similarity between assertoric content and proposition, a similarity which is so big that often assertoric content and proposition are not separated, they are mingled into one concept. The difference is that, one could say, the proposition is an assertoric content made into an object, a logical object in your theory. Saying that it is made into a logical object is the same as saying that you begin making assertions of the form

$$A \text{ is a proposition}$$

For the notion of proposition we have already the explanation given in connection with the BHK-interpretation, Brouwer–Heyting–Kolmogorov interpretation, namely that a proposition is an expectation, or intention, in Heyting's terms—which he got from Husserl via Oskar Becker, as is nowadays well known—and on the other hand, we have Kolmogorov's term, task, if I use task as the translation from his *Aufgabe* in German. That is the way the notion of proposition is fixed in constructive logic. But now, if propositions are just assertoric contents made into objects in your theory, then it seems reasonable to take this to be the explanation also of what an assertoric content is. So we define an assertoric content to be an expectation, or intention, in Heyting's terms, or task, in Kolmogorov's terms. That is my novel suggestion here, that we take this as the way of fixing the notion of content.

There remains the mood. The mood in general is simply the kind of speech act. I mentioned a few different kinds a moment ago, but today I only deal with two of them: the assertoric mood and the request mood.

If we take the notion of assertion first, then that can be defined immediately as soon we have introduced the assertoric mood, even without explaining it semantically. An assertion is simply something of the form

$$\vdash C$$

If you let the assertoric mood operate on a content, then that is what we call an assertion. By this I mean that, it is quite dangerous to utter something of this form, because just by uttering it, you are making an assertion. People will take you as making an assertion and take you to task if you do not agree to the rules for making assertions that we all are subject to, just as when you make a promise: a promise is a promise if you have given the promise, and you cannot get out of that by any means.

That is what an assertion is, but nothing has been said so far—I have explained the content, but I have not explained semantically the assertoric force. What determines, to my mind, the assertoric force is laying down the conditions under

which you have the right to make an assertion, which in the case of assertion is that you know how to fulfil, or perform, the content C. The trouble with the BHK-terminology is that expectation and intention go together with fulfil, and when you come to task, it is not perfect in English with fulfil, to my mind, but perform is the preferred verb. I think I will allow myself to say fulfil here in order not to have to duplicate the verb all the time.

So, the condition under which you have the right to make an assertion, $\vdash C$, is that you, that is, the speaker, knows how to fulfil C. I will not make any difference between knowing how to do something and being able to do something. That is a discussion by itself, and if I remember it correctly, Ryle, who first brought up the concept of knowledge how, he identified also knowledge how with ability, so I will stick to that. This assertion condition, which I have just given, is what fixes the meaning of the assertion sign, the assertive mood.

Now let us turn to the request mood. It is simplest to begin with the rules, because the explanation is visible directly from the rules. The rules that involve request are these, that if someone has made an assertion, then you may question his assertion, the opponent may question his assertion,

(Req1) $$\frac{\vdash C}{\underset{\text{may}}{?\vdash C}}$$

This is an example of a rule where we have a may. The other rule says that if we have the assertion, $\vdash C$, and it has been challenged, then the assertor must execute his knowledge how to do C. We saw what that amounted to in the two Ranta examples. I will write this schematically, that the assertor will continue by asserting zero, one, or more assertions—we have two in the existential case—so I will call that schematically C',

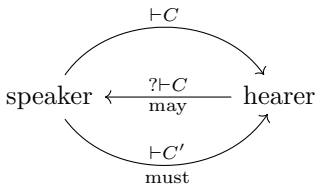
(Req2) $$\frac{\vdash C \qquad ?\vdash C}{\underset{\text{must}}{\vdash C'}}$$

In (Ex), C' consists of $a : A$ and $B(a)$ true, and in (Dis), C' is either A true or B true.

These are the two request rules. With Gentzen's terminology of introduction and elimination, where introduction is where some operator is occurring in the conclusion, and elimination is where it occurs in the premiss, one could say that (Req1) is the request introduction rule and (Req2) is the request elimination rule. The important novelty now is that the deontic modality in (Req2) is not may any longer, just as before, but must.

These rules immediately show what—the first of them shows what is the condition under which you have the right to make the request. The condition is that a previous judgement has been made, and you are challenging that judgement. The other rule shows what is the effect, or consequence, of making a request, namely of compelling the assertor to execute his knowledge, that is, to put his knowledge how to do C into practice. I think that by showing these rules, and explaining them in the way I have just done, there is nothing more that needs to be said by way of an explanation of the request mood.

Five more minutes roughly, so then, maybe, I will just show another way of displaying this structure, this back-and-forth structure, if you want, between the speaker and the hearer:



You have the first act of assertion, $\vdash C$, then the answer, which is the request by the hearer, and then the speaker is obligated to answer. This is just another picture to symbolize the logical rules here.

What is the most interesting with this? As for myself, I think it is the fact that we get now in addition to the ontological layer of logic, which deals with the content and its inner structure, and in addition to the epistemological layer of logic, which deals with assertion and inference, or reasoning, if you want—it does not matter if you say reasoning or inference or, in mathematics, demonstration: it amounts to the same. So we have these epistemological notions of assertion and inference, but what has appeared now is also deontic notions, which we are not used to thinking of as being at the base of logic. The way they come in is in saying something about how the knowledge how that is embodied in an assertion is manifested, and that knowledge how gets its manifestation in this dialogical game here between the assertor and the requester.

Perhaps I could symbolize it like this:

is	ontic
can	epistemic
may must	deontic

We have the modal auxiliaries—can, may, and must—of which may and must are deontic, and can is epistemic. Since they are auxiliary verbs, they have to operate on some main verb, and it is clear what the first main verb is in logic, namely the copula. The copula sits in the content, as in the example

$$A \text{ is a proposition}$$

The analysis of the copula belongs definitely to the ontic level of logic, so the novelty is the addition of this deontic underpinning of the two already existing layers of logic, the ontological and the epistemological layer, or ontic and epistemic layer. In 2006 I used on one occasion the title *The two layers of logic*, and the conclusion from this now is that there is actually, at the very bottom, a third level, namely the deontic level of logic.

There is a minute left, so let me say something about this in relation to what we call deontic logic and which has been in place since Mally—Ernst Mally was a pupil of Meinong at Graz—introduced the first systems of deontic logic in the 1920s, 1926 I think, and which was then taken up by Von Wright in Finland in '51. What I

have done here, is it deontic logic or is it not? Well, it is clearly not at all like what we call deontic logic following Mally and Von Wright, because in their system of deontic logic you make assertions, so it is always the question of assertions, but the deontic notions, they sit in the content. This means that deontic logic itself—its assertions have an epistemic character, and hence one could say that, in deontic logic, the deontic notions get subordinated somehow to the epistemic conception of logic, whereas in what I have done here, I have used these two deontic notions systematically, and they do not sit inside the content, that is quite clear.

Where do they sit? Well, they sit in the conclusions of the new rules of inference, or rules of interaction if you prefer. They sit in front of the conclusion in these rules, and that is a quite different place as compared with deontic logic. That is why it seems to me appropriate to say that these deontic notions are what you arrive at at the most basic level when you start from high up and get successively down to more and more basic concepts.

These deontic notions of may and must, or permission and obligation, where do we ordinarily say that they belong? Well, they belong to deontology, to use the term that was introduced by Bentham in the early 1800s. Deontology was Bentham's way of rendering duty ethics, I mean a Latin way of rendering duty ethics, and because of the correlativity of duties and rights, nowadays it is not so popular perhaps to speak about only duty ethics, better to speak about duty-and-right ethics. Anyway, the area which is based on the notions of rights and duties is deontological ethics, which means that, at its very root, logic is based on something which belongs to the area of ethics.