Epistemic assumptions: are they assumed to be backwards vindicated or forwards vindicable?

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Note. This is a transcript of a lecture given by Per Martin-Löf on 6 September 2019 in Leiden at a symposium in honour of Göran Sundholm on the occasion of his retirement. The transcript was prepared by Ansten Klev and has been slightly edited by the author. The abstract of the lecture can be found at the end of this document.

Göran Sundholm has introduced a new concept in logic, that of an epistemic assumption. Epistemic assumptions were there, of course, in many places: whenever you validate a rule of inference that might be under suspicion, you are making those assumptions. For instance, in my book on intuitionistic type theory with Giovanni Sambin, there are many, many such cases: many, many epistemic assumptions that are made, but no special name for them. It is Göran who introduced this name, and, as is so often the case, by so doing, by introducing a name, suddenly you see them all over the place, and how they are connected with other things in logic, which perhaps otherwise you would not have seen.

This occurred originally in 1997, in a paper called 'Implicit epistemic aspects of constructive logic', and there are, since then, several other places. The last one that I know of is from 2018 in a paper called 'The neglect of epistemic considerations in logic: the case of epistemic assumptions', where the epistemic assumptions are even in the title of the paper.

Since they are called epistemic assumptions, it is clear that epistemic is there to distinguish them from ordinary assumptions, ontic assumptions if you want, of the kind that we all know who have learned natural deduction. So let me just remind you of this, that if we have a proposition—and I am going to take directly the semantic approach: Gentzen and Prawitz would say formula, but we know that formulas are the formalistic counterparts of propositions, so I will say directly proposition A—then we may assume it. But here we should be careful, because if a proposition is defined by its truth conditions, then it is clear that we are not assuming the truth conditions: we are assuming the truth conditions to be fulfilled, which means that we are really assuming the proposition to be true,

$$A \text{ true}$$

although that is never written out in natural deduction. In natural deduction, you just have A, but in type theory, for instance, you have A true even as a formal notation of the language itself.

It is also a bit simplified to say that we just write A true when we make an assumption, because when we discharge assumptions, we have to have a label for the assumption that we discharge, a label that becomes bound in an implication introduction, for instance, and it is much more logical, especially with knowledge of the Curry–Howard correspondence, to introduce those labels of the assumptions already when you make them, and not only when you discharge them. So instead of A like this, we assume A and introduce a label,

$$\overset{x}{A}$$

For those labels, Gentzen and Prawitz use numbers, and that is a possible choice of labels, but from the Curry–Howard correspondence, we know that they function exactly like variables, so I am using this notation rather. Then the correspondence with constructive type theory is very clear: you just change that notation into

$$x : A$$

by introducing an explicit symbol for the copula.

So these are ordinary ontic assumptions, and I referred here to Gentzen and Prawitz, but of course we know that assumptions were made already in Greek logic, because reductio ad absurdum is impossible unless you can assume something and derive a contradiction from it. So the assumptions were there already with the hypothetical proofs.

Now let us go over to the epistemic assumptions. The simplest way of characterizing them is to say that the difference from an ordinary assumption is that now we no longer assume a proposition to be true, but instead we assume a judgement, or an assertion, to be known. So now instead of A true we will have a judgement, or an assertion, and we assume it to be known,

$$J \text{ known}$$

Concerning assertion and judgement, I do not want to dwell on that: it does not matter for this talk if I choose one or the other. What for me has been the natural choice has been to use judgement when you see yourself as in continuity with Bolzano, Brentano, Frege, and Husserl and other members of the Brentano school, because they all used the term Urteil, whereas if you are taking the viewpoint of speech act theory, then I find it more natural to use the term assertion. I think I will use assertion primarily in this talk. So an epistemic assumption is something of this form.

Now, to understand the real meaning of epistemic assumptions, you have to show the context in which they appear: you have to find out how they work, so to say. The place, as I have already mentioned, where they occur is when you are validating an inference or a rule of inference. An inference or a rule of inference looks something like this:

$$\text{(Inf)} \qquad \frac{J_1 \ \dots \ J_n}{J}$$

If you want to validate that, you begin by saying: assume the premisses, that is J_1, $\dots$, J_n, to be known, and under those assumptions you have to try to make the conclusion evident to yourself, or if you want, you have to try to get to know the conclusion. Here I have used a single schematic letter for assertions, or judgements, but by definition, an assertion, or judgement, is of the form $\vdash C$. So (Inf) necessarily looks like

$$\text{(Inf}'\text{)} \qquad \frac{\vdash C_1 \ \dots \ \vdash C_n}{\vdash C}$$

There is no difference whatsoever between writing it as (Inf) or (Inf′). The latter just makes it explicit that the assertion sign is the outermost sign of the assertion.

Now one could think that it is not more complicated than this, and that I ought not to say much more about it, because it seems fairly clear. But there is a problem with this, and that has to do with the sense of known here. Known has an ambiguity which I will take care of by introducing, in a very unimaginative and dry way,

$$\text{known}_1 \qquad \text{known}_2$$

The definition of what an epistemic assumption is is not sufficiently clear before we have disambiguated it, and as I have written already in the abstract, I think, the outcome will be that it is known$_2$ which is in question in the case of epistemic assumptions.

What are these two senses of known, known$_1$ and known$_2$? Known$_1$ is the simplest one to formulate, namely: an assertion, or judgement, is known in sense one if it is known demonstratively, and that is the same as saying that it is demonstrated, it has been demonstrated,

$$\text{known}_1 = \text{demonstrated}$$

where I take it that we all know what a demonstration is, although, towards the end of the talk, if there is time, I will actually say something about the concept of demonstration, but not at this point.

It is more difficult to find good formulations of what it is for an assertion, or judgement, to be known in sense two. What is known is still an assertion, or judgement, and the first formulation of known$_2$ is that it is an assertion whose content is known to be true. Here I am using Frege's standard terminology: Frege uses truth about judgemental contents, and we know how closely that is related to the very notion of assertion, because Frege in *Begriffsschrift* introduced the notation that I am using here, that was the beginning of his logic, and in his very last paper, 'Gedankengefüge', he actually abandons the assertion sign and writes instead C is true. It is true in this sense which I have in mind here.

Starting from this Fregean formulation, I now take the notion of truth to be analysed in terms of intention and fulfilment. So then we simply replace true by fulfillable: an assertion whose content is known to be fulfillable.

Finally, we have a third formulation where constructivity comes in explicitly, namely, the constructivist interprets 'whose content is known to be fulfillable' as 'whose content the speaker knows how to fulfil', which means that here the knowledge is required to be manifestable, in Dummett's terms. This I take to be the constructive meaning of known$_2$.

$$\begin{aligned} \text{known}_2 &= \text{whose content is known to be true} \\ &= \text{whose content is known to be fulfillable} \\ &= \text{whose content the speaker knows how to fulfil} \end{aligned}$$

You could say that these two steps are there in order to give to known$_2$ its constructive interpretation, and then there is also a natural way of making the distinction between the first step and the second step. The first step only consists in identifying truth with fulfillability of an intention, and that is a step that was taken in phenomenology by Husserl, and which Heyting came to benefit from via Husserl's assistant Oskar Becker. There has been an explicit influence on intuitionism from phenomenology, and here we see part of that: the identification of truth with fulfillability of an intention. So one could say that we have a phenomenological step, and then we have a more explicitly constructivist step.

My view is that there is needed now a little bit of systematic development in order to clarify this further. A heading for this systematic development could be

Correctness of assertion and validity of inference

We have two terms coming in here, namely correctness, which is tied to assertion, and validity, which is tied to inference. Concerning joining correctness to assertion, I do not think I need to say very much: both Göran and I have used in many, many places the term correctness of assertion, and the first to use it systematically were Bolzano and Brentano, with richtiges Urteil, so it is well established. And, of course, the notion of validity of inference is extremely well established, so one could—and I think I will—systematically use the words in this way. But an observation that I made during the preparation of this talk is that, really, they are interchangeable: you could just as well speak of correctness of inference as validity of inference, or in the other direction, you could just as well allow yourself to speak about validity of assertions and judgements, in which case you would have validity in both cases. What is essential here is that we have a normative term, a value term, a value concept that we apply both to assertion and to inference, and if we think of correctness as our primary way of making such an evaluation, then why not use correctness in both places? So it is enough with one word. Well, there are two concepts, of course, in the sense that, if you speak of correctness of assertion and correctness of inference, it is two notions of correctness. But they modify what they modify in the same way, so to say, and we could do just as well with only validity or only correctness.

This little systematic development here has to begin by saying what an assertoric content is. There I have originally taken inspiration from Heyting and Kolmogorov, because Heyting used the terms expectation and intention, and Kolmogorov used the term task, and my proposal is simply to define an assertoric content as an intention. One could use any of the half a dozen slightly different synonyms for intention that there are: purpose, end, etc. But since intention is what was used originally by Husserl and what was taken over by Heyting, I have settled for intention. So an assertoric content C we should think of as an intention. An assertion is then defined simply as what you get by prefixing the assertion sign to such an intention. This is by definition what an assertion is.

Now we come to the notion of correctness. How is correctness defined? That means that I have to make explicit what is nowadays naturally referred to as the correctness condition for assertion, and by so doing, I give what Williamson has baptized the knowledge account of assertion. Williamson has suggested that we should think of assertion as being defined by giving the condition under which an assertion is correct. What is that condition? Again we have two steps. The first formulation is: the condition for an assertion to be correct is that the speaker knows the content to be true. That is the way Frege and Williamson have it: Frege's account is a knowledge account in Williamson's terminology. Then we can replace true by fulfillable, and then we have the second step where we replace what we have here by 'the speaker knows how to fulfil the content'.

$$\begin{array}{rcl} \text{correct assertion} & = & \text{the speaker knows the content to be true} \\ & = & \text{the speaker knows the content to be fulfillable} \\ & = & \text{the speaker knows how to fulfil the content} \end{array}$$

Here of course it is essential that I have fixed the notion of content in the way I did as an intention, so that it makes sense to speak about fulfilling the content. Again this is as before, namely that we start from the classical formulation—knows the content to be true—and then we take two steps from there, first the phenomenological step, where we replace truth by fulfillability, and then the second more specifically constructive step.

I think this is enough about the notion of correctness of assertion, and I can go over to the notion of validity of inference. Here is an inference:

$$\frac{\vdash C_1 \ \ldots \ \vdash C_n}{\vdash C}$$

We are posing ourselves the question, Is this correct or not? Now I am in a bit of trouble here, since I have not made up my mind whether I am going to use valid or correct, any one of them will do, but why not be as traditional as possible here and use valid in the case of inference: correct for assertion, valid for inference. Then we have to give the analogue of this condition for inference. It could read as follows, that the condition for an inference to be valid is that, when the premiss assertions have been correctly made, then it is also correct to proceed to make the conclusion assertion.

This could now be given a more condensed formulation by using the terminology of preservation. We are used to speaking of preservation of truth from premisses to conclusion in an inference, and it is this sense of preservation that I am making use of here. So then we could say that an inference is valid if it preserves correctness of assertion from the premisses to the conclusion. Since an assertion is correct if and only if it is known, this amounts to the same as saying that knowledge is preserved from the premisses to the conclusion. In terms of the assertoric contents rather than the whole assertions, it means that knowledge of truth is preserved from the premiss contents to the conclusion content: in validating the inference, we start by assuming that the premiss contents are known to be true, and under those assumptions we have to get to know the truth of the conclusion content as well. It is natural to call this preservation of knowledge of truth from premiss contents to the conclusion content.

Observe now the difference with the usual formulation of validity of inference, when we say that truth is preserved from the premisses to the conclusion. That is correct for consequence, and since it is customary not to make the distinction between consequence and inference, one puts it in that way. That makes it correct, at least, but it is not inference, it is consequence. Whereas now that Göran and I and others have made a strenuous attempt to get understanding for the idea that there is a fundamental difference between consequence and inference, we have another place now where it becomes crucial, namely that we have no longer preservation of truth, but we have preservation of knowledge of truth in inference, preservation of truth only in consequence.

If we take the further step to the specifically constructive formulation, then we would have, in the case of inference, preservation of knowledge how to fulfil the assertoric content: we begin by assuming that we know how to fulfil $C_1, \dots, C_n$, and then we have to get to know how to fulfil C.

Then you see we are at the very point where the notion of epistemic assumption was born. When you speak about validity of inference, and you have some particular inference or inference rule that you consider, and if you look at the form of such a validation, it begins—as I said at least twice a minute or two ago—you begin by assuming that the premisses are known. That means that if we want to get clear about whether it is known in sense one or known in sense two that should be there in the definition of epistemic assumption, then it is here that we have to look, and the question is, Have I assumed the premisses to be known in sense one, or have I assumed them to be known in sense two? The answer should be clear already: when validating an inference, what you begin by assuming is that the premiss assertions have been correctly made. That is what I said when defining validity of inference. I assume that the premiss assertions have been correctly made, which means constructively that it is known how to fulfil their respective contents. This is the same as assuming that the premiss assertions are known in sense two rather than known in sense one. Remember, known in sense one was just demonstrated, and we are not assuming that the premisses have been demonstrated: we are assuming it known that $C_1, \dots, C_n$ are all true, in traditional terms, and then that is reformulated constructively in this way. So it is known in sense two, and not known in sense one, which is the appropriate notion of known in the definition of epistemic assumption.

I chose these very colourless terms known$_1$ and known$_2$ in order not to prejudge matters, but actually we have excellent terminology for this in Kant's table of the modalities of judgement. Kant had this famous table with four different groups, with three items in each group, and one of the groups was modality:

Modality
problematic
assertoric
apodictic

Now, known in sense one, that was, precisely, demonstrated. Since apodeixis is demonstration, apodictic is known in sense one. And known in sense two, that is the ordinary notion of being known for a judgement, in Kant's terminology, so assertoric can be identified with known in sense two,

$$\begin{aligned} \text{apodictic} &= \text{known}_1 \\ \text{assertoric} &= \text{known}_2 \end{aligned}$$

So now we have an excellent terminology, especially if we use the term judgement rather than assertion: one is apodictic, and the other is assertoric.

In what remains of the talk I will basically discuss how these two notions of known are related to each other, known_1 and known_2. I guess that should be more or less clear already, namely that if something has been demonstrated, if it is known in sense one, then it follows that we know its content to be true, phrasing it classically, which is, precisely, known in sense two,

$$\text{known}_1 \rightarrow \text{known}_2 \tag{S}$$

If an assertion is apodictic, that is, if it has been demonstrated, then it is correct to make it as an assertion.

Now I want to elaborate on this a bit and write it in another way. Known_1 is demonstrated, so I am saying of something which is known in sense one, which is always an assertion of the form $\vdash C$, so I am saying that it is demonstrated, and from that follows that it is correct to assert it,

$$\text{dem} \vdash C \;\rightarrow\; \text{cor} \vdash C$$

This is the same implication as (S) and has an obvious similarity to the soundness property in standard metamathematical treatments of propositional or predicate logic. In propositional or predicate logic we have the soundness theorem, which says that if—a formula in that case, because it is metamathematics—if a formula is derivable, then it is true,

$$\text{der}(F) \rightarrow \text{true}(F)$$

Here truth is Tarski truth.

So it seems that in this very dry formulation (S) we have something which is very fundamental when you spell it out in a better way, namely an analogue of the soundness property, which is now not on the metamathematical level, where you have the ordinary soundness theorem. Also, it is not on the propositional level, where you have replaced formulas by propositions, but it is rather on the level of judgements and their correctness that you have this property that if something has been demonstrated, then it is correct to assert it.

Let me finish then—next to finish, at least—by giving the argument for (S). Since we are on a purely conceptual level, this cannot be established by a mathematical proof, as we establish things in metamathematics. But that does not mean that we cannot say something to convince ourselves of it.

We have the two notions of demonstratedness and correctness, so the question is how they are defined. How do we define demonstrated, and how do we define correct? Correct I have already spoken about, but demonstrated, How do we define it? In a standard verbal formulation I would say that a demonstration is a formal derivation from principles which are valid. So if something is formally derived by means of valid principles, then it is demonstrated. If we spell that out, it could look like this:

$$\frac{\text{dem} \vdash C_1 \;\ldots\; \text{dem} \vdash C_n \qquad \text{val}\left(\dfrac{\vdash C_1 \;\ldots\; \vdash C_n}{\vdash C}\right)}{\text{dem} \vdash C} \tag{D}$$

If the inference rule is valid, and if $\vdash C_1, \ldots, \vdash C_n$ are all demonstrated, then also $\vdash C$ is demonstrated. This is a rule for which I have now used a notation that makes it look like an inference rule, but it is on the metalinguistic level, where we speak about inference, not in the language, and it is this rule which is meaning determining for demonstrated.

Then we have on the other hand, with correct instead of demonstrated:

$$\text{(V)} \qquad \frac{\text{cor} \vdash C_1 \;\ldots\; \text{cor} \vdash C_n \qquad \text{val}\left(\dfrac{\vdash C_1 \;\ldots\; \vdash C_n}{\vdash C}\right)}{\text{cor} \vdash C}$$

Whereas (D) was defining for dem, this rule (V) is defining for what validity is, because validity is defined as preservation of correctness from premisses to conclusion. But if the meaning of dem is given by (D), and correctness satisfies the same clauses as does dem, then that allows us to conclude that dem is at least as strong as cor. So it is simply the idea that we are used to from the soundness property in ordinary first-order logic courses, that truth propagates step by step following the formal derivation. Here we do not have that formal derivation, but we have the rule (D) instead, and we have the propagation still.

I have not said a word about vindicate, appearing in the title of the talk, because it is not necessary to use the word vindicate. I just wanted, by using it, to show that vindicate has an ambiguity which exactly fits the ambiguity between known_1 and known_2. The ambiguity that I have in mind is formulated by the *Oxford English Dictionary* in the following way. If you look under 'vindicate' or 'vindication'—it does not matter which one you choose, so let me choose 'vindicate'—you have sense three, which is

> To clear from censure, criticism, suspicion, or doubt, by means of demonstration; to justify or uphold by evidence or argument.

Here the term demonstration is even explicitly used. Sense four of vindicate, on the other hand, is

> To assert, maintain, make good, by means of action, esp. in one's own interest; to defend against encroachment or interference.

So it is to assert or make good by means of action. In the terms that I have been using we may say that vindication in sense three is demonstration and vindication in sense four is manifestation of your knowledge, which is what you insist upon constructively: if something has been demonstrated, then it should be possible to manifest the knowledge that you have acquired through the demonstration. So it is just a chance luck that—maybe it is something deeper—vindicate and vindication happen to have precisely this ambiguity that is involved in known_1 and known_2.

Abstract

In a paper from 1997, entitled Implicit epistemic aspects of constructive logic, Göran Sundholm introduced the term epistemic assumption for the assumptions with which the semantic (contentual) justification of a formal rule of inference invariably begins. The question to be discussed is whether these epistemic assumptions are assumed to be demonstrated or merely to be correctly (properly) asserted in the semantic (contentual) sense. I shall favour the second alternative.