

**ARE THE OBJECTS OF PROPOSITIONAL ATTITUDES
PROPOSITIONS IN THE SENSE OF PROPOSITIONAL AND
PREDICATE LOGIC?**

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NOTE. This is a transcript of a lecture given by Per Martin-Löf in Geneva on 19 December 2003. The transcript was prepared by Göran Sundholm using a draft made by Giuseppe Primiero. Emendations have later been made by Ansten Klev, who also prepared this L^AT_EX version.

The first problem with the title of my talk is the ambiguity in the term object when we speak of the objects of propositional attitudes: object is ambiguous between what is often called the intentional object, or grammatical object, if you analyse complete sentences grammatically, and the real object of the attitude. If the attitude is that of fearing that this winter will be warm, the object in the sense that I mean is

that this winter will be warm,

or

this winter to be warm,

if you prefer the accusative and infinitive construction. On the other hand, there is also the real object of my fear, namely the warmness of this winter, or the possible warmness of this winter, which I hope will not take place. This is a well-known distinction, and if you prefer to mark it terminologically, then you can use the term content instead of object when what you have in mind is the intentional object and not the real object.

Were I to explain the concept of propositional attitude and its content to a mathematician, I would have said something like this. Consider an example like the four-colour conjecture, namely the conjecture that every planar map can be coloured by four colours. That was conjectured, I think, already in the 19th century. Several people claimed it, but it turned out that they had to withdraw their claims, because errors were found in their demonstrations. At this stage, when it was still a conjecture, there might have been people who wished the four-colour conjecture to be true. For instance, someone printing maps might very well have hoped this, because he could then make do with just four colours, and the printing would be simplified. So it is certainly possible to wish that every map can be coloured by four colours. Other people might at this stage have feared that every planar map could be coloured by four colours, for instance the person who gave the very nice proof of the five-colour conjecture, very perspicuous proof, and one could have hoped that that would have been the final solution—hoped, but it turned out that it was not. Then eventually, in 1976, it was again claimed, or asserted, by Appel and Haken that a planar map can be coloured by four colours, and they were able to ground

their assertion, unlike the people who were around before. It is this grounding of the assertion that made it correct for them, unlike the previous people, to assert this.

What this example shows is that we can have one and the same content, or object, namely

that every planar map can be coloured by four colours,

or, with the accusative and infinitive construction,

every map to be four-colourable,

which is the object of many different attitudes: assertion in the first place, hope, fear, doubt, and so on.

In this example I kept the content fixed and varied the attitude. There is of course also the possibility of varying the content. I could change the content into the content that I used before,

that this winter will be warm,

or

this winter to be warm.

Some people may hope this, people who prefer the summer, for instance. Some people may fear this, such as those who like skating, and so on. There may even be people who assert that this winter will be warm, although it is very difficult to imagine that anyone at the present stage of meteorology could have a good basis for such a claim, so he would most likely have to withdraw it. Here we have another content being the object of the same varying attitudes as in the previous example.

What I hope to have made clear with these examples is that we have a very fundamental logical structure here, a structure that we may symbolize, and that was symbolized perhaps for the first time by Searle in the 1960s, in this way:

$$F(p)$$

Here F is the symbol for the kind of propositional attitude, and F is chosen because it stands for Frege's term *Kraft* in English, force. The letter p is the preferred symbol after Russell for proposition—it is what Searle called the propositional content. If you think the analysis of a complete sentence proceeds from without, which I certainly think myself as a logician, then this is indeed the most fundamental logical structure, because this is the outermost structure of a complete sentence.

Of course, this structure has been observed by many people, so let me make a list on the blackboard (Figure 1, next page). I put Frege first on the basis of the *Begriffsschrift*, because it has the analysis of assertion, or judgement, into two components, the content and the assertoric force, right at the beginning. Until the end of his life, Frege apparently considered this as maybe his most fundamental logical discovery. I put him first here on the basis of the date of the *Begriffsschrift*, but we must remember that he did not have the concept of force at that time. The concept of force comes only in 1906, which is already after Husserl's *Logical Investigations*. Frege then remarks on the *Ablösung der behauptenden Kraft*—what do you say, the dissociation of the assertoric force—from the content of a judgement,

Frege	Kraft	Inhalt Sinn
Husserl (LU)	Qualität	Materie
Husserl (Ideen)	Setzungscharakter thetischer Charakter	Sinn(eskern)
Russell	propositional attitude	proposition
Bally	modus	dictum
Hare	neustic	phrastic
Stenius	functional component mood	sentence radical descriptive content
Austin	illocutionary force	locutionary meaning
Searle	illocutionary force	propositional content

FIGURE 1

but he did not have much discussion about other attitudes, other forces, until much later.

Then we have Husserl's distinction between the quality and the matter of an act—act defined as intentional experience—in the *Logical Investigations*, which is kept in all of his later works. In *Ideen*, the terminology changes, and he rather speaks of the positing character and the sense, or the kernel sense, but the distinction is the same as in the *Logical Investigations*. Apparently, he thought that the older terminology was too closely tied to Brentano's descriptive psychology, and he wanted his own terminology.

What I used in the title of my talk is Russell's terminology: propositional attitude, and for the content there is no doubt, I mean, the attitude is called propositional precisely because Russell thought that the content of the attitude is a proposition.

Then going more in the linguistic direction, we have a distinction made by Charles Bally, who was the successor of Saussure here in Geneva. In his book *Linguistique générale et linguistique française* he starts by making the distinction between the mood, or the modus, and the dictum of a sentence, between the mood and what is said in the sentence. Mood here of course refers primarily to the mood in the grammatical sense, but I would take the view that the kind of attitude, or the kind of speech act, of which I have given several examples, is to be considered as the logical mood, rather than the grammatical mood, which means that when we pass from the logical analysis of a sentence to its grammatical form in one of our usual languages, then this logical mood will be reflected in the grammatical mood that you will use in expressing it. Beginning in Latin, though disappearing in the modern languages, the mood you use in these that-clauses that I have used to express a content, for instance, that every planar map be four colourable, is the subjunctive mood—subjunctive, of course, precisely because in the that-clause the facticity is not yet stated. So this is Bally's distinction between modus and dictum.

Hare's phrastic is of course just the dictum, and neustic is from “nod of assent”, which means that the neustic is the force in Frege's terminology.

In the speech act philosophy of the 1950s and '60s there was an outburst of interest in this, to give the fundamental structure of speech acts. I want to mention

in particular Stenius. He actually had two dichotomies here. On the one hand, he had the functional component and the sentence radical, the F above being the functional component and the p being the sentence radical. On the other hand, he spoke of mood and descriptive content. Why did he have these two dichotomies? Well, clearly he thought both syntactically and semantically, so he thought of the first of these as a syntactic distinction and the second as the corresponding semantic distinction.

Then we are in speech act philosophy proper with Austin. We have Austin's term illocutionary force, and for the content, at least in *How to do Things with Words*, he uses the term locutionary meaning. Searle took over the term illocutionary force, but he changed locutionary meaning into propositional content, essentially the Russellian term.

I began with Frege and Husserl, but if you think of the origin of this fundamental structure, although Husserl knew *Begriffsschrift*, it was certainly not that which was his primary source of inspiration. It was Brentano's classification of psychic phenomena into *Vorstellungen*, *Urteile*, and *Gemütstätigkeiten* which made the strongest impression on him. On the other hand, if we go back to Brentano, he referred, in *Vom Ursprung sittlicher Erkenntnis*, to Descartes for this distinction. The passage is in the third Cartesian meditation, where Descartes says,

when I will, fear, affirm, or deny, I always, indeed, apprehend something as the object of my thought,

here we thus have the object of the thought,

but I also embrace in thought something more than the representation of the object; and of this class of thoughts some are called volitions or affections, and others judgments.

We have here a clear separation between the kind of thought—volition, affection or judgement—and the object of thought, and that is precisely the structure that I have tried to make clear here.

Now, for Descartes, as well as for Brentano and Husserl, I think, this was primarily the thought structure, that is, the structure of the cogito, the cogitatio-cogitatum structure, to begin with, but when we went on in this list, eventually it was turned into a linguistic distinction, namely the distinction in a sentence between its mood, logical or grammatical mood, and its content. This fits, of course. You may say that the structure that I am considering here, considered as a logical structure, logico-grammatical structure, is nothing but the linguistic turn of the cogito. It is very natural, if you think that there is a correspondence between thought and language, that there should be a correspondence between the structures that we find when we analyse thought and those that we find when we analyse language. Indeed, this particular structure can be arrived at in either of these two ways, but it is one and the same structure.

This much about the concept of propositional attitude and the object, or content, of a propositional attitude. What is the other half of my title? I put something there which should be as unambiguous as at all possible. We have something we call propositional logic and predicate logic, and in predicate logic we have propositional functions, and of course we could not say what we need to say in dealing with

those without having a term for the things that are combined by means of the logical operations. If A is such a thing, then $\sim A$ is such a thing, and similarly with the connectives and the quantifiers—if we have a function defined over some individual domain which takes such things as values, then we can form the quantified things. So we have to have some term for this, and of course the standard term is proposition, I mean, it is propositional logic, so it would be very unwise to invent a new term for this.

I could have said something else in posing the question of my title, namely speaking instead of the bearers of truth and falsity. The things that you combine by means of the logical operations, they are also the bearers of truth and falsity, because on the classical conception, a proposition is defined precisely by having one of these, truth or falsity. Aristotle’s formulation was: something that has truth or falsity in it. So certainly, the bearers of truth and falsity are the same as the things that we combine by means of the logical operations.

The question then is this: are these the same as the objects, or contents, of propositional attitudes? Or do we have an ambiguity in the term proposition here, because the objects of propositional attitudes are propositions in one sense, and the bearers of truth and falsity combined by means of the logical operations are propositions in another sense?

Let us begin with early Frege. In the *Begriffsschrift*, how does he write modus ponens, for instance? In this way:

$$\frac{\begin{array}{l} \vdash B \\ \vdash A \end{array}}{\vdash B}$$

This immediately shows something. Let us first rewrite this in modern notation,

$$\frac{\vdash A \supset B \quad \vdash A}{\vdash B}$$

I already said that the fundamental structure of the judgement, or assertion, is this:

$$\underbrace{\begin{array}{c} \text{force} \quad \text{content} \\ \downarrow \quad \downarrow \\ \vdash \quad A \end{array}}_{\text{assertion, judgement}}$$

We have the whole assertion, or judgement, and its two components of assertoric force and content. We see then immediately that the A and B here cannot be anything but judgemental contents, judgeable contents as Frege said, and, since we have an implication here, this implication operates on these contents. So, early Frege definitely took the logical operations to operate on contents, and this is a view that he stuck to all his life. In his very last published paper, “Gedankengefüge”, he did the same thing. It looks like this:

$$\frac{\begin{array}{l} (\text{if } A, \text{ then } B) \text{ is true} \\ A \text{ is true} \end{array}}{B \text{ is true}}$$

This is the way he wrote modus ponens then. There is no difference in sense here. The only difference is that he writes out the implication sign as if-then, and he

writes out $\vdash A$ as A is true. What, then, of my question here? In Frege, as much in “Gedankengefüge” as in *Begriffsschrift*, the logical operations operate on contents, judgemental contents.

Then we have Frege of the middle period, which is to say Frege of the time of *Grundgesetze* and the immediately surrounding years of its publication, which is essentially the 1890s. Frege’s view at that time was more complicated, as was first observed by Russell in Appendix A of *The Principles of Mathematics*, which had the title “The Logical and Arithmetical Doctrines of Frege”.

In § 2, already, of *Grundgesetze*, Frege defines what he means by a thought:

The sense of the name of a truth value I call a *thought*.

Den Sinn des Namens eines Wahrheitwertes nenne ich Gedanken.

That is the definition of thought in *Grundgesetze*. Maybe I can then go directly to § 32. Frege defines thought as the sense of a name of a truth value, and thought, of course, corresponds to content in the *Begriffsschrift*. Is that at all compatible, one may ask, with his earlier view of judgeable content and with his late view in “Gedankengefüge”, where he calls that thought: can I take an attitude to the sense of the name of a truth value? Clearly not, I mean, I cannot do that anymore than I can take an attitude towards the sense of a numerical expression. A numerical expression refers to a number, but I cannot assert it. It is not the sort of thing that I can assert, or wish, or fear, or anything. In Frege’s standard formulation, the sense of a name of a truth value is the mode of presentation of a truth value. Can I take an attitude towards a truth value presented in a particular way? No, clearly not, because it does not say anything. What I can take an attitude towards is the thought that it refers to the true. That I can certainly have an attitude towards: I can assert it, of course, in the first place, or wish it, or fear it, or whatever it is.

There is thus a clear difficulty here. I mean, there is a clear inner conflict in Frege’s thought here, if you compare the middle Frege with the early and late view, and that comes out most clearly if you look at the crucial § 32 in *Grundgesetze*:

Every such name of a truth value *expresses* a sense, a *thought*.

Jeder solche Name eines Wahrheitwertes drückt einen Sinn,
einen Gedanken aus.

This is just a restatement of what I quoted before from § 2. The name of a truth value expresses a sense, and that sense of the name of a truth value he calls thought. This is in complete agreement with what he said in § 2. The next sentence, where he refers to truth conditions of a certain kind, has no problems. But then comes the really crucial sentence:

The sense of this name, the *thought*, is this, that these conditions are fulfilled.

Der Sinn dieses Namens, der Gedanke, ist der, dass diese Bedingungen erfüllt sind.

The sense of this name, the thought—that is the first half of the sentence, that is perfectly all right, exactly how he defined thought in § 2: the sense of this name is the thought. But then he says that this thought, the sense of the name, is that the truth conditions are fulfilled. A step is taken here: the step from the sense of

the name of a truth value to the thought that the reference of that name is the true. There is a difference between the sense of the name of a truth value and the thought that it is the true rather than the false that it denotes, that it refers to.

To my mind, this is the precise point where there is a difficulty in Frege. If we stick to the idea of the thought being the sense of the name of a truth value, then we need three components rather than two components in this fundamental structure: A is now the sense of the name of a truth value, which is the same as a thought in the sense of *Grundgesetze*, but, as I said, that does not by itself form something to which I can attach assertoric force. I first have to say that the truth condition that defines A is fulfilled, or that the thought A refers to the true, on Frege's conception in the 1890s: it is that which can form the content of an assertion. Let me write that out explicitly here,

$$\begin{array}{c} \text{thought in the sense} \\ \text{of } \textit{Grundgesetze} \\ \downarrow \\ \vdash \underbrace{A \text{ true}} \\ \text{thought in the sense} \\ \text{of late Frege} \end{array}$$

Expressed in one way, A true means that A refers to the true. With the other formulation that Frege used, it means that A 's truth condition is fulfilled, or A 's truth conditions are fulfilled—I think Frege used the plural. What we have here is a judgeable content, judgemental content, or assertoric content, if you want, in the sense of *Begriffsschrift*, and that corresponds to thought as used in the famous paper *Der Gedanke* from 1918, so a very late paper. (There is no doubt, I think, that content in the sense of *Begriffsschrift* is thought in the sense of *Der Gedanke*.) And then we have the assertoric force, \vdash .

This, of course, is not how Frege himself discussed it, that is quite clear. I am rather showing that there is an inner tension in the view, and I am bringing that out, while making something consistent out of it, by introducing this third ingredient, third component, here in the fundamental structure.

Russell was the first to note this tension, in 1903 already (§ 477, p. 503):

We have first a propositional concept, next its truth or falsity as the case may be, and finally the assertion of its truth or falsity.

So we have the propositional concept, next, its truth or falsity as the case may be, and the result is what Russell, apparently at that stage following Meinong, whose *Über Annahmen* he had just read, called an assumption,

$$\begin{array}{c} \text{propositional} \\ \text{concept} \\ \downarrow \\ \underbrace{A \text{ true/false}} \\ \text{assumption} \end{array}$$

It is not so strange that he called this an assumption, because when we make an assumption in ordinary reasoning as formalized in natural deduction, it is clear that if the proposition is defined by its truth condition, as we traditionally say, then we cannot assume just a truth condition: we assume a truth condition to be fulfilled and cannot just assume the condition. On the other hand, when we make an assumption, we do not assume that we know that it is true: we just assume that it is true. When we make a natural-deduction step, what we establish is the

truth of the final proposition, that the final proposition is true, provided that all the hypotheses are true. That is what we establish, and what we know as a result of the natural deduction is precisely that. We clearly do not assume that we know that the hypotheses are true, but we just assume that they are true.

This simple consideration is, I think, convincing enough in the form that I have given here. There are three components. There is the proposition as defined by a truth condition, that is the A here, and then we have A true, which says that the truth condition is fulfilled, and then all of this,

$$\vdash A \text{ true}$$

where you add the assertoric force, expresses the knowledge that the truth condition is fulfilled. Repeating Russell again: we have first the propositional concept, next its truth or falsity as the case may be, and finally the assertion of its truth or falsity. This, to my mind, is the correct analysis here, which Frege in a sense had, but in another sense did not have. To make his view in the 1890s coherent, you have to introduce this third component in the fundamental structure.

Now I want to compare the classical and constructive conceptions. Classically, a proposition is thought of as being defined by its truth condition, even in the extreme way that Frege had, as a name of a truth value. What happens if we turn from this classical conception to the constructive conception instead? More precisely, what is the type-theoretical analysis of this fundamental structure? We then have to change, first of all, the concept of proposition. Constructively, a proposition is defined by its proof conditions, rather than its truth conditions. Proof conditions are laid down by displaying the possible forms that a canonical proof of the proposition can take. You explain conjunction, for instance, by saying that a canonical proof of a conjunction, $A \& B$, is a pair consisting of a proof of A and a proof of B , and similarly for all the other logical operations. This is what replaces the classical definition of a proposition.

Truth is defined now in terms of the underlying notion of proof, by saying that a proposition is true if there exists a proof of it. The notion of existence that comes in here needs its own explanation, since it is not the notion of existence captured by the existential quantifier, but another notion of existence. It is explained by saying that what you must know in order to have the right to judge a proposition to be true is that you have in your possession a proof of it, a proof which may in general be non-canonical, that is, merely be a method or program for producing a canonical proof.

What I just said here is not very different from saying that the notion of truth is defined by the following rule, or that the following rule is meaning determining for the concept of truth,

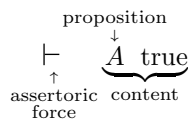
$$\frac{a \text{ is a proof of the proposition } A}{A \text{ is true}}$$

or in the form of notation that is used in type theory,

$$\frac{\vdash a : \text{proof}(A)}{\vdash A \text{ true}}$$

I explain the concept of truth by saying what I must know in order to have the right to judge a proposition to be true, and the answer is that I must know a proof of it. The form of judgement $\vdash A$ true is just an abbreviated way of saying $\vdash a : \text{proof}(A)$, I am just not showing you explicitly what the proof is that I have constructed. I can phrase that equivalently by saying that this rule is meaning determining for the form of judgement $\vdash A$ true. Just as we say with Gentzen that the introduction rules for the logical operations determine their meaning, I can allow myself to say that the meaning of the form of judgement $\vdash A$ true is determined by this rule.

The remarkable thing here is that, although Frege in the 1890s had an entirely classical conception of the concept of proposition as the sense of a name of a truth value, the three-component analysis that he had—it is not quite correct to say that he had it, but it is what he would have needed in order to make his conception entirely coherent—is vindicated also constructively, albeit under an entirely different interpretation of the notion of proposition and of what it means for a proposition to be true. We have exactly the same structure now even constructively. We have the assertoric force, we have the content, and we have the proposition,



I have redefined what a proposition is, I have redefined what truth means, but otherwise it is exactly the same structure.

What does this say about the question that was the title of my talk? Are propositions the objects, or contents, of propositional attitudes? The content of a propositional attitude—that of assertion in this case—is A true, so it is not a proposition alone that makes up this content. You have to add the truth operator to the proposition in order to get the assertoric content. The answer to the question that was the title of my talk is therefore an emphatic *no*. You could probably have guessed that from the beginning: there is a difference between the notion of proposition and the notion of assertoric content.

I would like to use the remaining ten minutes or so to relate this distinction to some good terminologies. The first is the terminology of Satzradikal, whether you want to translate it as sentence radical or as proposition radical. The term Satzradikal was introduced by Wittgenstein in § 23 of the *Investigations*, and it has been heavily exploited by Stenius, whom I referred to earlier, and it is a very good term, to my mind. So let us look at it.

In grammar, we have the notion of a complete sentence. What is a complete sentence? Well, it is all of this,

$$\vdash A \text{ true}$$

That is the Satz, and then Wittgenstein introduced the term Satzradikal in order to have a term for the part A true, so the sentence radical is what I generally have called the content here. In propositional and predicate logic we are thus not dealing with sentence radicals or proposition radicals. Rather, this A , which is what we call a proposition in propositional and predicate logic, is a radical of a radical. We have to take away also the truth operator before we get to it. So a proposition in

the sense of propositional and predicate logic is a Satzradikalradikal, whereas the content of a judgement is a Satzradikal.

Another very good terminology that we may profitably use to express the conclusions that I have reached here is Dummett's terminology, originally from the Frege book: assertoric content versus ingredient sense. Dummett was led to this distinction when considering non-intended interpretations of logic—on the one hand, many-valued interpretations, and on the other hand, possible-worlds interpretations. He said (pp. 446-447),

... , we must distinguish, ... , between knowing the meaning of a statement in the sense of grasping the content of an assertion of it, and in the sense of knowing the contribution it makes to determining the content of a complex statement in which it is a constituent: let us refer to the former as simply knowing the *content* of the statement, and to the latter as knowing its *ingredient sense*.

Then he convincingly argues that when you have non-intended interpretations, like many-valued interpretations and possible-worlds interpretations, you must distinguish between these two, because the assertoric content is not determined compositionally from the parts of a sentence or proposition. Rather, you have something, which he calls the ingredient sense, that is determined compositionally, and then you get from the ingredient sense to the assertoric content by means of a special operation. In the case of many-valued logic you would, and you know how it goes, you—I think we all know this, both in the case of many-valued logic and in possible-worlds semantics.

I should say that Dummett also said that there is no such distinction in the case of the intended interpretation, because then ingredient sense and assertoric content collapse, and Dummett, being the Fregean he is, refers to Frege for this. In Frege there is no distinction between these, there is indeed a collapse, hence so far this is correct. But the outcome of my discussion here is precisely that even in the intended interpretation, we do not have a conflation, or collapse, between ingredient sense and assertoric content. Rather, we can, I propose, use this excellent terminology that already exists to say in a new way what I have already said in at least two different ways before: we have what is determined compositionally by induction on the build-up of the structure of A , namely, the ingredient sense, and then we need to apply the truth operator, or this grammatical truth particle, to it in order to get the assertoric content, and then, if we want the whole assertion, we must in turn apply the assertoric force. Dummett's terminology is indeed excellent here for expressing the conclusion that I have reached.

Then, maybe one more comment. We also have the terminology used so much in the *Tractatus* of states of affairs as contrasted with facts. The question is now: what in this picture is most appropriately called state of affairs and what fact? This is a question now, since the two-component structure that we are used to has been replaced by this three-component structure, so we have, so to say, a distinction between meaning and truth on two levels. On one level, the proposition A is the meaning, and A true is the truth of it, and on another level it is the content, A true, that is the meaning and the assertion, $\vdash A$ true, that is the truth of it. Remember

that Frege used A is true as a way of expressing the latter, and the notion of truth that he then uses is expressed by the assertion sign, whereas in the form of content A true we have another notion of truth. This is a source of endless confusion. We are really dealing with two notions of truth here that we must clearly keep apart if we are not going to end up in trouble. And it is the same then with the notions of state of affairs and fact, because they appear on two levels.

If you think of how the notions are used in the *Tractatus*, it is clear that it is on the level of propositions and their truth: propositions match states of affairs and obtaining states of affairs correspond to true propositions, so a state of affairs is a fact if it is true in this sense.

But look now at natural language. According to a standard definition, the indicative mood is the fact-stating mood. If you want to use the terms state of affairs and fact in such a way that they agree with ordinary use, it is therefore not the proposition A that should be called the state of affairs, but rather the content A true: it is the assertoric content that is a state of affairs. Then it is the assertion

$$\vdash A \text{ true}$$

which expresses that A true is a fact, and that is how Frege used “ist eine Tatsache” in the *Begriffsschrift*. He read the assertion, not only as “ A ist wahr”, but also as “ A ist eine Tatsache”. Moreover, a state of affairs is naturally expressed by a that-clause, precisely as a content is.

My conclusion concerning these two terms is therefore that we have to make a decision. Are we going to use them on the propositional level, or are we going to use them on the higher level, so to say? If you want to have an agreement with how they are used in natural language, and also in grammar in connection with the indicative mood, which is of course the grammatical counterpart of the assertoric force, then, indeed, you should identify state of affairs with content, assertoric content, and fact with asserted content.